

Object Oriented FDTD Method

Design Documentation

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Abstract

This report describes the object oriented methodology used in the design of a finite difference time domain (FDTD) code. The aim is to have an easily expandable code base which can be fitted to different purposes. In this report FDTD with uniaxial perfectly matched layer (UPML) and dispersive materials (Lorentz) is Fix the abstract...

I. OVERVIEW OF FDTD

A. General FDTD Expressions in 3D

Let's start with Maxwell's equations:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{E} = -\mathbf{M} - \frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\mathbf{J} = \mathbf{J}_{source} + \sigma \mathbf{E} \quad \sigma = \text{siemens/meter} \quad (3)$$

$$\mathbf{M} = \mathbf{M}_{source} + \sigma^* \mathbf{H} \quad \sigma^* = \text{ohms/meter} \quad (4)$$

For linear, isotropic, non-dispersive and lossy materials we have the following set of equations:

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \mathbf{E} - \frac{1}{\mu} (\mathbf{M}_{source} + \sigma^* \mathbf{H}) \quad (5)$$

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon} \nabla \times \mathbf{H} - \frac{1}{\epsilon} (\mathbf{J}_{source} + \sigma \mathbf{E}) \quad (6)$$

Looking at the x, y and z components of the equations above result in the following mambo-jambo:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - M_{source,x} - \sigma^* H_x \right) \quad (7)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - M_{source,y} - \sigma^* H_y \right) \quad (8)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - M_{source,z} - \sigma^* H_z \right) \quad (9)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_{source,x} - \sigma E_x \right) \quad (10)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\epsilon} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - J_{source,y} - \sigma E_y \right) \quad (11)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_{source,z} - \sigma E_z \right) \quad (12)$$

Let's now discretize the 3D space to get the difference equations for the fields. One will be done in detail, the rest follows by duality. Just keep track of the subscripts.

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_{source,x} - \sigma E_x \right) \quad (13)$$

$$\begin{aligned} \frac{E_x|_{i+1/2,j,k}^{n+1} - E_x|_{i+1/2,j,k}^n}{\Delta t} = \frac{1}{\epsilon} \left(\frac{H_z|_{i+1/2,j+1/2,k}^{n+1/2} - H_z|_{i+1/2,j-1/2,k}^{n+1/2}}{\Delta y} - \frac{H_y|_{i+1/2,j,k+1/2}^{n+1/2} - H_y|_{i+1/2,j,k-1/2}^{n+1/2}}{\Delta z} \right. \\ \left. - J_{source,x}|_{i+1/2,j,k}^{n+1/2} - \sigma \frac{E_x|_{i+1/2,j,k}^{n+1} - E_x|_{i+1/2,j,k}^n}{2} \right) \end{aligned} \quad (14)$$

Collecting terms gives the following update equation for E_x :

$$\begin{aligned} E_x|_{i+1/2,j,k}^{n+1} = \frac{1 - \frac{\sigma_{i+1/2,j,k}\Delta t}{2\epsilon_{i+1/2,j,k}}}{1 + \frac{\sigma_{i+1/2,j,k}\Delta t}{2\epsilon_{i+1/2,j,k}}} E_x|_{i+1/2,j,k}^n + \frac{\frac{\Delta t}{\epsilon_{i+1/2,j,k}}}{1 + \frac{\sigma_{i+1/2,j,k}\Delta t}{2\epsilon_{i+1/2,j,k}}} \times \\ \left(\frac{H_z|_{i+1/2,j+1/2,k}^{n+1/2} - H_z|_{i+1/2,j-1/2,k}^{n+1/2}}{\Delta y} - \frac{H_y|_{i+1/2,j,k+1/2}^{n+1/2} - H_y|_{i+1/2,j,k-1/2}^{n+1/2}}{\Delta z} \right. \\ \left. - J_{source,x}|_{i+1/2,j,k}^{n+1/2} \right) \end{aligned} \quad (15)$$

For the other field components, derivations are almost identical, the only difference being that the symbol names and subscripts change. Trying to be as careful as possible, here are the

other update equations. Beware, there might be bugs, so it's best to check the equations below by hand just in case...

$$E_y|_{i,j+1/2,k}^{n+1} = \frac{1 - \frac{\sigma_{i,j+1/2,k}\Delta t}{2\epsilon_{i,j+1/2,k}}}{1 + \frac{\sigma_{i,j+1/2,k}\Delta t}{2\epsilon_{i,j+1/2,k}}} E_y|_{i,j+1/2,k}^n + \frac{\frac{\Delta t}{\epsilon_{i,j+1/2,k}}}{1 + \frac{\sigma_{i,j+1/2,k}\Delta t}{2\epsilon_{i,j+1/2,k}}} \times$$

$$\left(\frac{H_x|_{i,j+1/2,k+1/2}^{n+1/2} - H_x|_{i,j+1/2,k-1/2}^{n+1/2}}{\Delta z} - \frac{H_z|_{i+1/2,j+1/2,k}^{n+1/2} - H_z|_{i-1/2,j+1/2,k}^{n+1/2}}{\Delta x} \right. \quad (16)$$

$$\left. - J_{source,y}|_{i,j+1/2,k}^{n+1/2} \right)$$

$$E_z|_{i,j,k+1/2}^{n+1} = \frac{1 - \frac{\sigma_{i,j,k+1/2}\Delta t}{2\epsilon_{i,j,k+1/2}}}{1 + \frac{\sigma_{i,j,k+1/2}\Delta t}{2\epsilon_{i,j,k+1/2}}} E_z|_{i,j,k+1/2}^n + \frac{\frac{\Delta t}{\epsilon_{i,j,k+1/2}}}{1 + \frac{\sigma_{i,j,k+1/2}\Delta t}{2\epsilon_{i,j,k+1/2}}} \times$$

$$\left(\frac{H_y|_{i+1/2,j,k+1/2}^{n+1/2} - H_y|_{i-1/2,j,k+1/2}^{n+1/2}}{\Delta x} - \frac{H_x|_{i,j+1/2,k+1/2}^{n+1/2} - H_x|_{i,j-1/2,k+1/2}^{n+1/2}}{\Delta y} \right. \quad (17)$$

$$\left. - J_{source,z}|_{i,j,k+1/2}^{n+1/2} \right)$$

Enough? Well, there's also the magnetic field... Hold on a bit more, almost over...

$$H_x|_{i,j+1/2,k+1/2}^{n+1/2} = \frac{1 - \frac{\sigma_{i,j+1/2,k+1/2}^*\Delta t}{2\mu_{i,j+1/2,k+1/2}}}{1 + \frac{\sigma_{i,j+1/2,k+1/2}^*\Delta t}{2\mu_{i,j+1/2,k+1/2}}} H_x|_{i,j+1/2,k+1/2}^{n-1/2} + \frac{\frac{\Delta t}{\mu_{i,j+1/2,k+1/2}}}{1 + \frac{\sigma_{i,j+1/2,k+1/2}^*\Delta t}{2\mu_{i,j+1/2,k+1/2}}} \times$$

$$\left(\frac{E_y|_{i,j+1/2,k+1}^n - E_y|_{i,j+1/2,k}^n}{\Delta z} - \frac{E_z|_{i,j+1,k+1/2}^n - E_z|_{i,j,k+1/2}^n}{\Delta y} \right. \quad (18)$$

$$\left. - M_{source,x}|_{i,j+1/2,k+1/2}^n \right)$$

$$H_y|_{i+1/2,j,k+1/2}^{n+1/2} = \frac{1 - \frac{\sigma_{i+1/2,j,k+1/2}^*\Delta t}{2\mu_{i+1/2,j,k+1/2}}}{1 + \frac{\sigma_{i+1/2,j,k+1/2}^*\Delta t}{2\mu_{i+1/2,j,k+1/2}}} H_y|_{i+1/2,j,k+1/2}^{n-1/2} + \frac{\frac{\Delta t}{\mu_{i+1/2,j,k+1/2}}}{1 + \frac{\sigma_{i+1/2,j,k+1/2}^*\Delta t}{2\mu_{i+1/2,j,k+1/2}}} \times$$

$$\left(\frac{E_z|_{i+1,j,k+1/2}^n - E_z|_{i,j,k+1/2}^n}{\Delta x} - \frac{E_x|_{i+1/2,j,k+1}^n - E_x|_{i+1/2,j,k}^n}{\Delta z} \right. \quad (19)$$

$$\left. - M_{source,y}|_{i+1/2,j,k+1/2}^n \right)$$

$$\begin{aligned}
H_z|_{i+1/2,j+1/2,k}^{n+1/2} = & \frac{1 - \frac{\sigma_{i+1/2,j+1/2,k}^* \Delta t}{2\mu_{i+1/2,j+1/2,k}}}{1 + \frac{\sigma_{i+1/2,j+1/2,k}^* \Delta t}{2\mu_{i+1/2,j+1/2,k}}} H_z|_{i+1/2,j+1/2,k}^{n-1/2} + \frac{\frac{\Delta t}{\mu_{i+1/2,j+1/2,k}}}{1 + \frac{\sigma_{i+1/2,j+1/2,k}^* \Delta t}{2\mu_{i+1/2,j+1/2,k}}} \times \\
& \left(\frac{E_x|_{i+1/2,j+1,k}^n - E_x|_{i+1/2,j,k}^n}{\Delta y} - \frac{E_y|_{i+1,j+1/2,k}^n - E_y|_{i,j+1/2,k}^n}{\Delta x} \right. \\
& \left. - M_{source,z}|_{i+1/2,j+1/2,k}^n \right)
\end{aligned} \tag{20}$$

B. UPML Implementation

C. Lorentz Implementation

1) Parameter Fitting to Experimental Data For a Given Frequency Range:

II. DESCRIPTION OF FIELD TELEPORTATION METHOD

III. OBJECT ORIENTED METHODOLOGY

IV. DESCRIPTION OF OBJECT IMPLEMENTATIONS

A. Cell Class

B. DielectricECCell Class

C. PECBrick Class

D. PMLCell Class

E. LorentzCell Class

V. VALIDATION STRUCTURES