Qubit-Photon Interactions in Waveguides

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Outline

Overview of Quantum Computing Platforms
   Requirements from a Quantum Computer
   Circuit QED Examples
   Quantum Optics Examples

Photon-Qubit Interactions in Waveguides
   Perfect Single Photon Mirrors
   Nonlinear Scattering of Two Photon Packets
   Resonance Fluorescence From a Qubit
   Modal Dispersion & Formation of Atom-Photon Bound Modes
   Two-Photon Scattering in a Dispersive Waveguide
   Multi-Qubit Numerical Time Evolution Studies
   Pauli Z Gate Implementation

Summary
Why Quantum Information Processing?

- Algorithms exist that enable very fast computation using entangled states
  - Factoring large numbers
  - Searching big data
  - Solving a large number of equations
- Secure communications
- Simulate quantum many-body systems
Stages in Quantum Computing

- Lowest two levels: physical layer
- Physical limits on coherence time $\rightarrow$ error correction\(^1\)
- Use of quantum error correction to achieve quantum memory with longer coherence than physical subsystems (not yet done)

Circuit QED Implementations

- Josephson junction based qubits
- Connections by microwave transmission lines
- Cooled down to millikelvin temperatures
- Qubit energy levels and their coupling to resonators are controlled with external signals

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Formation of Gates

- Gates can be formed by tuning energy levels of qubits\(^3\)

- Or by external microwave pulses which excite certain transitions among different energy levels\(^4\)

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Commercial Quantum Annealing Processor: D-Wave

- Specialized system, implements quantum annealing only\(^5\)
- Degree of ‘quantumness’ being discussed
- Dec 8, 2015 “[Quantum annealing] is more than \(10^8\) times faster than simulated annealing running on a single core.”\(^6\)

Example hardware graph of \(12 \times 12\) units, each with 8 qubits, some are defective

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Quantum Optics Based on Nanophotonics

- Photonic crystal waveguide with two supported modes\(^7\)
- One mode used to trap ions
- Other mode used to communicate with ion based qubits

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Quantum Optics Based on Plasmonics

- Plasmonic wedge waveguide cavity\(^8\)
- Quantum dots are printed on the wedge
- External laser light used to pump quantum dot, emission into waveguide mode

Quantum Optics Based on Photonic Crystals

▶ Defect line in a photonic crystal forms a waveguide\(^9\)
▶ Quantum dot grown on In(Ga)As layer
▶ Input and output gratings used to couple light in and out

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Focus on Modeling of Physical Layer

- Gates do operations on ‘stationary’ qubits\(^\text{10}\)
- Results should be transferred by ‘flying’ qubits
- Interconnects are an integral part of the design
- Remainder of the talk on recent advances in understanding photon - qubit interactions in waveguides

QEC: quantum error correction
FT: fault tolerant

\(^{10}\)Van Meter and Horsman, Commun. ACM 56, 84–93 (2013).
System Details

- Qubit positioned in an anti-node of the waveguide mode

\[ |↑\rangle \quad |↓\rangle \quad \Omega \quad e^{-i\omega_k t} \quad e^{-i\omega_p t} \]

- Multi-mode Jaynes-Cummings Hamiltonian under RWA and dipole approximation

\[
H = \int dk \omega_k \hat{a}_k^\dagger \hat{a}_k + \frac{\Omega}{2} \sigma_z + g \int dk (\sigma_k^+ \hat{a}_k + \hat{a}_k^\dagger \sigma^-)
\]

- Linearized dispersion relation \((\text{for now})\)
Qubit as a Perfect Single Photon Mirror

- Hamiltonian re-written in real space
- Stationary state written as

\[ |E_k\rangle = \int dx \left[ \phi_{k,R}^\dagger(x)c_R^\dagger(x) + \phi_{k,L}^\dagger(x)c_L^\dagger(x) \right]|0\rangle + e_k \sigma^+|0\rangle \]

\[ \phi_{k,R}^\dagger(x) = \exp(ikx)\theta(-x) + t \exp(ikx)\theta(x) \]

\[ \phi_{k,L}^\dagger(x) = r \exp(-ikx)\theta(-x) \]

- Solve for \( H|E_k\rangle = E_k|E_k\rangle \)
- Transmission and reflection of single photon\(^{11}\)

More than one qubit doable via transfer matrix approach

Non-Linearity by Two-Photons

- Qubit cannot absorb two photons $\rightarrow$ non-linearity
- Complicated stationary states (Bethe ansatz)$^{12}$
- Formation of non-separable two-photon states

$$\langle k_2, R p_2, R | S | k_1, R p_1, R \rangle = t_{k_1} t_{p_1} [\delta_{k_1,k_2} \delta_{p_1,p_2} + \delta_{k_1,p_2} \delta_{p_1,k_2}]$$

$$+ \frac{1}{4} B \delta_{k_1+p_1,k_2+p_2}$$

$k_1 + p_1 - 2\Omega$ starts as 0 in (a) and increases in (b)–(d)

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What is Resonance Fluorescence?

- Excite atoms with a laser light (*coherent state*)
- Observe in an orthogonal direction
- Control signals are in coherent states, important to analyze qubit-coherent state interaction in waveguides

Mollow Triplets

- Light emitted from the atom has coherent (i.e. same freq) and incoherent components (i.e. different freq)
- High excitation intensity → Rabi oscillations → Mollow triplet

from Mandel, L. & Wolf, E.  
The $g^{(2)}$ Function in Resonance Fluorescence

$g^{(2)}(\tau)$ gives the probability of observing two photons with delay $\tau$ at the detector.

- Rabi oscillations at higher intensity $\rightarrow$ oscillations in $g^{(2)}$
- Anti-bunching as seen from $g^{(2)}(\tau = 0) = 0$

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Graph showing the $g^{(2)}(\tau)$ function with a peak at $\tau = 0$ and oscillations as $\tau$ increases.

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Resonance Fluorescence in Waveguides — Spectrum

- Scattering of a coherent state with varying intensity from a qubit\(^\text{13}\)

\[
\begin{align*}
    r_{\text{in}} & \quad e^{-ikt} \\
    \ell_{\text{out}} & \quad e^{-i\omega t} \\
    \Omega, \tau & \quad |\psi\rangle \\
    |g\rangle & \quad r_{\text{out}}
\end{align*}
\]

- Input-output formalism\(^\text{14}\) used, much faster analysis

Resonance Fluorescence in Waveguides — $g^{(2)}$

- Bunching for transmitted light
- Anti-bunching for reflected light
- Interference of radiated fields in forward/backward directions
Dispersive Modes of Waveguides — Tight Binding Case

Multi-mode Jaynes-Cummings Model

Tight-binding lattice dispersion \( \omega_k = -2J \cos k \), extension to other types of dispersion also possible

\[
H = \int_{-\pi}^{\pi} dk \omega_k a_k^{\dagger} a_k + \frac{\Omega}{2} \sigma_z + g \int_{-\pi}^{\pi} dk (\sigma^+ a_k + a_k^{\dagger} \sigma^-)
\]

Analyses will be made via the use of the resolvent

\[
G(z) = \frac{1}{z - H}
\]
Bound Modes

- We need the following matrix elements of the resolvent

\[ \langle \uparrow | G(z) | \uparrow \rangle \equiv G_1(z) \]
\[ \langle k \downarrow | G(z) | \uparrow \rangle \equiv G_2(z; k) \]
\[ \langle \uparrow | G(z) | k \downarrow \rangle \equiv G_3(z; k) \]
\[ \langle p \downarrow | G(z) | k \downarrow \rangle \equiv G_4(z; p, k) \]

- The poles of \( G_1(z) \) define *bound modes*, i.e. polaritonic modes, of the system\(^{15}\)

- There are two bound modes with energies \( \omega_{\pm} \) outside the free-photon energy range \((-2J, 2J)\)

\[ |\Psi_{\pm}\rangle = \sqrt{p_b} |\uparrow\rangle + \sqrt{p_b g} \int_{-\pi}^{\pi} dk \frac{|k\downarrow\rangle}{\omega_{\pm} + 2J \cos k} \]

Scattering matrix elements can be calculated via $T(z)$

$$\langle p \downarrow | S | k \downarrow \rangle = \langle p | k \rangle - 2\pi i \delta(\omega_p - \omega_k) \lim_{\eta \to 0^+} \langle p \downarrow | T(\omega_p - \Omega/2 + i\eta) | k \downarrow \rangle$$

It is easy to get $T(z)$ from $G(z)$ via the following relationships

$$T(z) = V + VG(z)V \quad \text{and} \quad G(z) = G_0(z) + G_0(z) T(z) G_0(z)$$
Feynman Diagram Representation

- From the Lippmann-Schwinger equation  
\[ T(z) = V + VG_0(z) T(z) \]  
we get  
\[ T(z) = \frac{1}{1 - VG_0(z)} V = V + VG_0(z) V + VG_0(z) VG_0(z) V + VG_0(z) VG_0(z) VG_0(z) V + \ldots \]

- The series representation leads to the Feynman graphs\(^{16}\)

\[ \equiv \langle p\downarrow |V G_0 V |k\downarrow \rangle \]
\[ \equiv \langle p\downarrow |V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V G_0 V |k\downarrow \rangle \]

\[ \equiv \langle p\downarrow |T |k\downarrow \rangle \]

Two-Photon Scattering in a Dispersive Waveguide

- Two-photon calculations require the following matrix elements

\[
\langle p^\uparrow | G(z) | k \uparrow \rangle \equiv G_5(z; p, k) \quad \langle p_1 p_2 \downarrow | G(z) | k \uparrow \rangle \equiv G_6(z; p_1, p_2, k)
\]

\[
\langle p^\uparrow | G(z) | k_1 k_2 \downarrow \rangle \equiv G_7(z; p, k_1, k_2) \quad \langle p_1 p_2 \downarrow | G(z) | k_1 k_2 \downarrow \rangle \equiv G_8(z; p_1, p_2, k_1, k_2)
\]

- \( G_6, G_7, G_8 \) can be written in terms of \( G_5 \)

- Analysis based on solvable Lee model of QFT

- We derive an integral equation for \( G_5 \)

\[
U(z; p, k) = \frac{1}{z - \omega_p - \omega_k} + g^2 \int_{-\pi}^{\pi} dp_i \frac{U(z; p_i, k)}{H(z; p_i)(z - \omega_p - \omega_{p_i})}
\]

- \( U \) function can be described in terms of Feynman graphs
Two photons lead to many possible diagrams
Test of Feynman Diagram Calculations — Bound Modes

- Free photon coming and scattering off of a bound mode
- Much better agreement as number of diagrams increased for large $g$
Test of Feynman Diagram Calculations — Free Modes

- Two-photon scattering calculated with Feynman diagrams (up) and independent numerical calculations (down)
- Initial state is two right going photons, final state is plotted on the left
- R: right, L: left, RR: two right going photons
Numerical Time Evolution Studies

Developed a numerical algorithm to evaluate the time evolution of multi-qubit systems with the Hamiltonian

$$H = -J \sum_{i=1}^{L-1} \left( a_{i+1}^\dagger a_i + a_i^\dagger a_{i+1} \right) + \sum_{s=1}^{n} \frac{\Omega_s}{2} \sigma_z^s + \sum_{s=1}^{n} \bar{g}_s \left( \sigma^+_s a_{x_s} + a_{x_s}^\dagger \sigma^-_s \right)$$

for an arbitrary initial state.\(^\text{17}\)

Time Evolution of $|↑↑⟩$ State

- Analytic calculations quickly get quite complicated
- Multi-qubit, multi-photon calculations are of interest for gate design
- Numerical time evolution via Krylov-subspace method
- $|Ψ(t)⟩ = e^{-iHt}|↑↑⟩$
- Excitation of a two-qubit bound mode, with a bouncing photon between qubits
Similar to the single qubit case, multi qubit systems support bound modes, too.\(^\text{18}\)

Unlike the single qubit case, some multi qubit bound states are within the propagating photon energy continuum range \(\rightarrow\) BIC

For two qubits we have\(^\text{19}\)

\[
|\psi_{n_{e,o}}^\pm\rangle = \mathcal{N} \left[ |\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle + \sum_x (-i)\bar{g} \frac{e^{ik_x|x+R/2|} \pm e^{ik_x|x-R/2|}}{\sqrt{4J^2 - \Omega^2}} |x\downarrow\downarrow\rangle \right]
\]

\[
\mathcal{N} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1 + \frac{\bar{g}^2 R}{4J^2 - \Omega^2}}},
\]


BIC Shapes

- Condition for BIC is $1 \pm e^{i k_\ast R} = 0$ and $\Omega = -2J \cos k_\ast$
- For $\bar{g} \ll 1$ we have $N^2 \approx 0.5$
DFS Based Logical Qubits

- One can associate the BIC with decoherence-free states\(^{20}\) from the Dicke model.
- There are quantum gate designs that make use of decoherence-free states (DFS)\(^{21}\).
- Two qubits make one *logical* qubit with

\[
|0\rangle \equiv |\downarrow\downarrow\rangle \\
|1\rangle \equiv |\Psi_{n_0}^-\rangle \approx \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]

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Pauli (-Z) Gate

- We use four physical qubits to get two logical qubits
- Initial state is \((|10\rangle + |01\rangle)/\sqrt{2}\)
- We apply a \(\pi\)-pulse to arrive at the final state \((|10\rangle - |01\rangle)/\sqrt{2}\)
- \(\Delta H_{12} = \frac{\Delta}{2}(\sigma_{z1} + \sigma_{z2})\)
- (set 1) \(R = 4, \bar{g} = 0.1, n_o = 1\)
  - (set 2) \(R = 4, \bar{g} = 0.5, n_o = 1\)
  - (set 3) \(R = 7, \bar{g} = 0.5, n_o = 1\)
  - (set 4) \(R = 7, \bar{g} = 0.5, n_o = 3\)
Summary

Waveguide integrated qubits are now technologically possible
Rich collection of effects arising from photon-qubit interactions in waveguides
- Particularly when waveguide dispersion is taken into account

Outlook
- Waveguide + qubits system is a promising simulation platform for many-body physics\textsuperscript{22}
- It seems also feasible to investigate topological order with synthetic dimensions excited in waveguide + qubits\textsuperscript{23}

\textsuperscript{22}Douglas, Nat Photon 9, 326–331 (2015).
Thank you for your attention.

Happy to share the slides if you are interested, email me at ekocabas@ku.edu.tr to get a copy.